

# Modern Trends in Structures Forming Systems Program

Heidelberg, August 7-11 2017

**Organizers:** Laurent Bétermin, Hans Knüpfer, Anna Marciniak-Czochra and Philipp Reiter.



RUPRECHT - KARLS -  
UNIVERSITÄT  
HEIDELBERG



## Contents

<b>1</b>	<b>Locations and Schedule</b>	<b>2</b>
<b>2</b>	<b>Abstracts for the talks</b>	<b>3</b>
<b>3</b>	<b>Abstracts for the poster session</b>	<b>10</b>

# 1 Locations and Schedule

The conference will take place at the Mathematikon, Im Neuenheimer Feld 205, Heidelberg, 6-11 August, 2017.

The conference will start on **Sunday the 6th** at **19:00** with a **reception**, in the common room of the Mathematikon, on the 5th floor.

The **talks** (50 min each, included questions) will take place in the **Hörsaal**, on the ground floor of the Mathematikon.

The **coffee breaks** will take place in the **Seminarraum C**, adjacent to the Hörsaal.

The **poster session and the buffet** will take place on Monday in Seminarraum C.

Please find below the schedule:

	Monday 7	Tuesday 8	Wednesday 9	Thursday 10	Friday 11
9:00	B. Niethammer	L. Bronsard	A. Stevens	W. Jäger	C. Melcher
9:50	B. Wirth	B. Schweizer	R. Schulz	A. Schlömerkemper	P. Dondl
10:40	<i>Coffee break</i>				
11:10	A. Capella	M. Bonacini	M. Röger	R. Ignat	M. Kruzik
12:00	<i>Poster session</i>	<i>Lunch</i>			
14:00	W.-P. Düll	A. Mikelic	<i>Excursion</i>	S. Bartels	
14:50	<i>Coffee break</i>			<i>Coffee break</i>	
15:20	M. Kyed	L. Chen		P. Pozzi	
16:10	G. Grün	J. L. Woukeng		H. Fritz	
19:00			<i>Social dinner</i>		

An excursion will be organized to **Abbey Stift Neuburg** by the famous **Philosophenweg**.

The social dinner will take place at **Kulturbrauerei Heidelberg AG, Leyergasse 6**, at 19:00.

There is a **participation fee of 10 euros**. Please notice that **all drinks** will be at your own expense.

## 2 Abstracts for the talks

### Sören Bartels (Universität Freiburg)

*Title:* Numerical solution of nonsmooth problems and application to optimal insulation

*Abstract:* Nonsmooth minimization problems and singular partial differential equations arise in the description of inelastic material behavior, image processing, and modeling of non-Newtonian fluids. The numerical discretization and iterative solution is often based on regularizing or stabilizing terms. In this talk we address the influence of such modifications on error estimates and the robustness of iterative solution methods. In particular, we present an unconditional stability result for semi-implicit discretizations of a class of singular flows and devise a variant of the alternating direction of multipliers method with variable step sizes. The results and methods are illustrated by numerical experiments for a problem of optimal insulation leading to a break of symmetry.

### Marco Bonacini (Universität Bonn)

*Title:* Self-similar solutions with time-dependent tails for Smoluchowski's coagulation equation

*Abstract:* Smoluchowski's coagulation equation is a nonlocal integral equation used to describe irreversible mass aggregation phenomena. A key question in the analysis of this model is whether the long-time behaviour of solutions is universal and described by special self-similar solutions. In this talk I will discuss some recent progress on this topic, valid for a class of rate kernels with homogeneity smaller than one, and for a class of kernels which are homogeneous of degree one. In both cases, I will present an existence result of a new one-parameter family of self-similar solutions with finite mass and time-dependent tails. Such solutions are characterized in terms of their precise asymptotic behaviour at infinity. This is joint work with Barbara Niethammer and Juan J.L. Velázquez.

### Lia Bronsard (McMaster University)

*Title:* Droplet phase in a nonlocal isoperimetric problem under confinement

*Abstract:* We begin with a variational model for the self-assembly of diblock copolymers under confinement, which takes the form of an isoperimetric problem which is both nonlocal and nonhomogeneous. That is, we seek minimizers in the form of characteristic functions of fixed volume. The energy consists of three competing terms, and minimizers should reduce their perimeter (as in the classical isoperimetric problem,) but also prefer spatial separation into disjoint components, which are confined by an attractive potential. We consider periodic configurations in the small volume fraction limit, in which one phase forms vanishingly small droplets in a sea of the complementary phase. Introducing a small parameter  $\eta > 0$ , which represents the radii of the droplets, we show that the minority phase splits into several droplets which converge to the maximum value of the confining potential, at an intermediate scale  $\eta^{1/3}$ . Isolating the droplets at the scale  $\eta^{1/3}$  requires a fine analysis of the blown-up problem, using concentration-compactness and the regularity properties of minimizers of the nonlocal isoperimetric problem on  $\mathbb{R}^3$ . This is joint work with Stan Alama, Rustum Choksi and Ihsan Topaloglu.

### Antonio Capella (Universidad Nacional Autónoma de México)

*Title:* Rigidity, quasiconvex envelopes and triple junctions in martensitic thin film

*Abstract:* In this talk we study pattern formation in martensitic thin films limits of bulk samples in a geometrically linear elasticity regime. We will present two extreme results. First, particular configurations where a rigidity results – including surface energy – may be obtained, that is where low energy configuration are given always by simple laminates. Second, we investigate energy relaxation for 3 rank-one connected wells by study their quasi convex envelope, and derive corresponding effective relaxed energies. In our analysis we show a remarkable relationship between the non existence of triple junctions and the quasiconvexification procedure.

**Li Chen (Universität Mannheim)**

*Title:* Diffusion-Aggregation Equations

*Abstract:* In this talk, I will give a short introduction on the rigorous derivation of the diffusion aggregation equations from many particle stochastic systems with different interaction potentials. Afterwards, an overview of the analytical results, which includes the global existence and finite time blow-up results on different kinds of diffusion aggregation equations.

**Patrick Dondl (Universität Freiburg)**

*Title:* The Effect of Forest Dislocations on the Evolution of a Phase-Field Model for Plastic Slip

*Abstract:* We consider a phase field model for dislocations introduced by Koslowski, Cuitino, and Ortiz in 2002. The model describes a single slip plane and consists of a Peierls potential penalizing non-integer slip and a long range interaction modeling elasticity. Forest dislocations are introduced as a restriction to the allowable phase field functions: they have to vanish at the union of a number of small disks in the plane. Garroni and Müller proved large scale limits of these models in terms of Gamma-convergence, obtaining a line-tension energy for the dislocations and a bulk term penalizing slip. This bulk term is a capacity stemming from the forest dislocations.

In the present work, we show that the contribution of the forest dislocations to the the viscous gradient flow evolution is small. In particular it is much slower than the timescale for other effects like elastic attraction/repulsion of dislocations, which, by a recent result due to del Mar Gonzales and Monneau is already slower than the time scale from line tension energy. Overall, this leads to an effective behavior like a gradient flow in a wiggly potential.

**Wolf-Patrick Düll (Universität Stuttgart)**

*Title:* Justification of the Nonlinear Schrödinger approximation for dispersive systems

*Abstract:* In order to describe the dynamics of oscillating wave packets in complicated dispersive evolutionary systems, the Nonlinear Schrödinger (NLS) equation can be formally derived as an approximation equation for the dynamics of the envelopes. To understand to which extent this approximation yield correct predictions of the qualitative behavior of the original systems it is important to justify the validity of the NLS approximation by estimates of the approximation errors in the physically relevant length and time scales. If the original systems are quasilinear, the justification of the NLS approximation is a highly nontrivial problem.

In this talk, we give an overview on the NLS approximation and its applications, for example, for modeling water waves, light pulses or spin waves, and prove the validity of the NLS approximation for typical examples of quasilinear dispersive systems.

**Hans Fritz (Universität Regensburg)**

*Title:* On the computation of harmonic maps by unconstrained algorithms based on totally geodesic embeddings

*Abstract:* e present an algorithm for the computation of harmonic maps, and respectively, of the harmonic map heat flow between two closed Riemannian manifolds. Our approach is based on the totally geodesic embedding of the target manifold into some Euclidean space. Totally geodesic embeddings allow to reformulate the harmonic map heat flow in a neighbourhood of the embedded target manifold. The reformulation has the advantage that the problem becomes unconstrained: Instead of assuming a priori that the solution to the flow maps into the target manifold this fact becomes a property of the solution to the extended flow for special initial data. This simplifies the discretization of the problem. Based on this observation, we propose algorithms for the computation of the harmonic map heat flow and of harmonic maps. We prove error estimates in the stationary case and present some numerical tests.

### **Günther Grün (Universität Erlangen-Nürnberg)**

*Title:* Thin liquid films influenced by thermal fluctuations: modelling, analysis, and simulation

*Abstract:* For liquid films with a thickness in the order of  $10^1 - 10^3$  molecule layers, classical models of continuum mechanics do not always give a precise description of thin-film evolution: While morphologies of film dewetting are captured by thin-film models, discrepancies arise with respect to time-scales of dewetting.

In this talk, we study stochastic thin-film equations which differ from corresponding deterministic equations by multiplicative noise inside an additional convective term. We present numerical simulations which indicate that the aforementioned discrepancies may be overcome under the influence of noise.

In the second part of the talk, we prove existence of almost surely nonnegative martingale solutions. Our analysis relies on spatial semi-discretization, discrete variants of energy and entropy estimates as well as on recent tools for martingale convergence.

The results have been obtained in collaboration with K. Mecke and M. Rauscher and with J. Fischer, respectively.

### **Radu Ignat (Université Paul Sabatier, Institut de Mathématiques de Toulouse)**

*Title:* One-dimensional symmetry of transition layers of vanishing divergence

*Abstract:* We focus on  $N$ -dimensional maps  $u$  of vanishing divergence defined on an infinite cylinder. They minimize a Modica-Mortola type functional with a nonnegative potential  $W$  under a boundary condition at infinity that forces  $u$  to make a transition between two zeros of  $W$ . We characterize the class of potentials  $W$  such that every global minimizer  $u$  is one-dimensional. Our method is based on calibrations adapted to divergence-free maps.

It is a joint work with Antonin Monteil (Louvain-la-Neuve).

### **Willi Jäger (Universität Heidelberg)**

*Title:* Mathematical Modelling and Analysis of Processes in Poroelastic Media - Applications in Medicine

*Abstract:* Modelling reactive flows, diffusion, transport and mechanical interactions in media consisting of multiple phases, e.g. of a fluid and a solid phase in a porous medium, is giving rise to many open problems for multi-scale analysis and simulation. The following processes are taken into account in this lecture:

- diffusion, transport, and reaction of substances in the fluid and the solid phase,
- mechanical interactions of the fluid and solid phase,
- change of the mechanical properties of the solid phase by chemical reactions,
- growth of the materials.

Using a scale limit, an effective model is derived, coupling the filtration flow, the mechanical deformation and the chemical reactions on the macroscopic level. A Biot-law is replacing the Darcy-law, which is used for non-deformable media.

Processes in biological tissues are discussed as applications and the following examples are presented:

- the swelling of cells caused by hypoxia, modelled and simulated, using a Biot-law for the flow inside cells,
- the biochemical and biophysical processes in the arterial wall relevant for its mechanical behaviour, for inflammation and disorders of the cardiovascular systems,
- the movement of a tooth in the alveolar bone under external forces,
- the mechanical behaviour of a ligament, a tissue connecting teeth with bone, modelled by a Biot-law.

The lecture is based on results obtained in co-operation with Andro Mikelic, Maria Neuss-Radu, Martina Kihn, Valeria Malieva.

**Martin Kružík (Czech Academy of Sciences)**

*Title:* Gradient-polyconvex materials

*Abstract:* Modern approaches to elasticity are based on the assumption that the first Piola-Kirchhoff stress tensor possesses a potential called stored energy density,  $W$ , which depends on the deformation gradient. Such materials are then called hyperelastic. If we additionally assume that external forces applied on a body are conservative, equilibrium equations of elasticity are formally Euler-Lagrange equations for minimizers of the elastic-energy functional. Existence of minimizers can be ensured if  $W$  is polyconvex, for instance. Polyconvexity also allows for physically realistic behavior of  $W$ , i.e., orientation-preservation of deformations and that  $W(F) \rightarrow +\infty$  if  $\det F \rightarrow 0$ . Nevertheless, many materials cannot obey polyconvex stored energy density. A prominent example are e.g. shape-memory alloys. A possible solution, often found in literature, is to assume that the stored energy density depends also on the second deformation gradient and is convex in it.

We show the existence of minimizers under weaker assumptions, namely, we make the energy density depend on gradients of nonlinear minors of the deformation gradients. Moreover, we outline some interesting properties of minimizers and a few applications to modeling of shape memory materials and plasticity.

This talk is based on a joint work with B. Benešová and A. Schlömerkemper (both from Würzburg).

**Mads Kyed (TU Darmstadt)**

*Title:* On  $L^p$  estimates for time-periodic solutions to parabolic boundary value problems of Agmon-Douglis-Nirenberg type

*Abstract:* A celebrated result of AGMON, DOUGLIS and NIRENBERG states that if an elliptic operator  $A$  satisfies the so-called complementing condition with respect to a number of boundary operators, then a solution to the corresponding boundary value problem satisfies an a priori  $L^p$  estimate. A similar result holds for the initial-value problem associated to the parabolic operator  $\partial_t - A$ . In my talk, I will consider the time-periodic boundary value problem related to the operator  $\partial_t - A$  and show a time-periodic version of the Agmon-Douglis-Nirenberg Theorem. I will present a simple proof based on Fourier multipliers and a Paley-Wiener theorem, from which the original elliptic case version of the theorem follows as a special case. Moreover, I will show that the estimates for the parabolic initial-value problem follows from the time-periodic version of the theorem, and thus argue that there is simple a one-fits-all approach to all three cases.

**Christof Melcher (Universität Aachen)**

*Title:* Topological solitons in chiral magnetism

*Abstract:* Chiral skyrmions are topological solitons occurring in magnets without inversion symmetry. In this talk I shall explain the analytical structure and variational consequences of chiral interactions responsible for the occurrence of new magnetic phases in the spirit of Ginzburg-Landau theory, and for the stabilization of magnetic skyrmions. New physics is predicted in the case of anisotropic chiral interactions, where skyrmions and antiskyrmions may coexist.

**Andro Mikelić (Institut Camille Jordan, Université Lyon 1)**

*Title:* Thermoporoelasticity via homogenization

*Abstract:* A derivation of the equations of the semi-linear thermoporoelasticity from the pore scale linearized fluid-structure and energy equations is undertaken. The starting point are the continuum mechanics thermodynamically compatible pore scale equations corresponding to realistic rock mechanics parameters. They are upscaled using the two-scale asymptotic expansions and a macroscopic model for the thermoporoelasticity is obtained. For the upscaled equations a Lyapunov functional (a generalization of Biot's free energy) is constructed and the well-posedness of the model is discussed. Possible applications to the long time intervals numerical simulations are pointed out. This is a work in collaboration with C.J. van Duijn (Darcy Center), M.F. Wheeler (Austin) and T. Wick (Ecole Polytechnique).

**Barbara Niethammer (Universität Bonn)**

*Title:* Instabilities and oscillations in coagulation equations

*Abstract:* Smoluchowski's classical mean-field model for coagulation is used to describe cluster formation and growth in a large variety of applications. The equation is nonlinear and nonlocal and involves a rate kernel that describes the microscopic details of the specific aggregation process. A question of particular relevance is whether the long-time behaviour is universal and described by self-similar solutions or traveling waves respectively.

In this talk I will focus on the case of rate kernels of homogeneity one which is a borderline case that separates mass conservation from the phenomenon of gelation. Formal arguments lead to the conjecture that for large times the coagulation equation can be seen as a regularization of the Burgers equation. In contrast to diffusive regularizations, however, we obtain phenomena such as instability of the constant solution or oscillatory traveling waves.

**Paola Pozzi (Universität Duisburg-Essen)**

*Title:* On finite element schemes for geometric flows coupled to lateral diffusion

*Abstract:* In this talk I will propose and analyze a semi-discrete finite element scheme for a system consisting of a geometric evolution equation for a curve and a parabolic equation on that evolving curve. More precisely, curve shortening flow with a forcing term that depends on a conserved field is coupled with a diffusion equation for that field. Such a system can be considered as a prototype for more complicated problems as they may arise in applications.

Next, new numerical results for systems involving the elastic flow will be discussed.

This is joint work with Björn Stinner.

**Matthias Röger (TU Dortmund)**

*Title:* Coupled bulk-surface reaction-diffusion systems and pattern formation

*Abstract:* In biological cells a tight regulation of processes in the cell and on the cell membrane is for many functions of the cell essential. We introduce different mathematical models in the form of coupled bulk-surface reaction-drift-diffusion systems and investigate in particular pattern forming properties.

**Anja Schlömerkemper (Universität Würzburg)**

*Title:* Towards an evolutionary model for magnetoviscoelasticity incorporating the Landau-Lifshitz-Gilbert equation

*Abstract:* Magnetoelastic materials have fascinating properties and a large variety of applications. Their mathematical modeling faces the fundamental issue that elasticity is phrased in Lagrangian coordinates whereas magnetism is phrased in Eulerian coordinates. We discuss a model that is completely phrased in Eulerian coordinates and takes microstructures of the magnetization into account. The model presented is a system of partial differential equations that contains (1) the incompressible Navier-Stokes equations with magnetic and elastic terms in the stress tensor obtained by a variational approach, (2) a regularized transport equation for the deformation gradient and (3) the Landau-Lifshitz-Gilbert equation for the dynamics of the magnetization. We will indicate the derivation of the model and will present results on the analytical properties of the system.

**Raphael Schulz (Universität Erlangen-Nürnberg)**

*Title:* Precipitation-Dissolution Model in Evolving Porous Media

*Abstract:* The talk deals with crystal dissolution and precipitation in a saturated porous media. This precipitation-dissolution process takes place on the surface of the solid matrix and changes the porosity of the porous media locally. Furthermore, the model allows changes in hydrodynamic parameters and hence impedes the flow. Using formal periodic homogenization, we derive an averaged model describing the process via Darcy's law and upscaled transport equations with effective coefficients provided by the evolving microstructure at the pore-scale. Assuming that the underlying pore geometry may be described by a single parameter, e.g. porosity, the level set equation locating the solid-liquid interface transforms into an ordinary differential equation for the parameter. We state significant analytical and algebraic properties of these effective parameters. A further objective of this talk is the analytical investigation of the resulting model. In a weak sense, unique solvability either global in time or at least up to a possible clogging phenomenon is shown.

**Ben Schweizer (TU Dortmund)**

*Title:* On waves in photonic crystals: radiation conditions and negative refraction

*Abstract:* We investigate the following transmission problem: a wave travels in free space and hits the boundary of a photonic crystal. Waves are described with Helmholtz equations, in the photonic crystal the Helmholtz equation has periodic  $x$ -dependent coefficients. Experiments show a multitude of interesting effects: (1) perfect reflection. (2) localized modes along the interface. (3) partial transmission with positive and/or negative refraction. We discuss the radiation conditions that must be imposed in this wave-guide problem, discuss a uniqueness result and present a numerical scheme.

We present joint work with A. Lamacz and T. Dohnal.

**Angela Stevens (Universität Münster)**

*Title:* Sorting phenomena due to attraction and repulsion

*Abstract:* In cellular systems as well as for biochemical reactions on cellular surfaces sorting phenomena play an important role. Often these processes are driven by diffusion, attraction and repulsion. A mathematical model for two species with such features will be analyzed and conditions for sorting will be classified. This is joint work with M.Burger, M. DiFrancesco, and S. Fagioli.

**Benedikt Wirth (Universität Münster)**

*Title:* A class of models for ramified transportation networks and corresponding phasefield approximations

*Abstract:* A small number of models for transportation networks (modelling street, river, or vessel networks, for instance) has been studied intensely during the past decade, in particular the so-called branched transport and the so-called urban planning. They assign to each network the total cost for transporting material from a given initial to a prescribed final distribution and seek the cost-optimal network. Typically, the considered transportation cost per mass is smaller the more mass is transported together, which leads to highly patterned and ramified optimal networks.

We present a joint framework for a broad class of transportation network models (including existing models) based on the geometric measure notion of flat 1-chains, which more easily allows to prove equivalence of various existing model formulations. In addition, based on this setting we present a phasefield approximation, which covers different models than recently developed phasefield approaches to transportation networks. Computational methods for transportation networks are only just emerging, and the phasefield methods are a first step towards simulating the complicated patterns of optimal networks.



**Jean Louis Woukeng (Université de Dschang)**

*Title:* Some Analytical and Numerical Methods for Multiscale Systems

*Abstract:* Most structures in nature and technology exhibit multiscale features both in space and time. In natural sciences, modelling and simulation have proven to be useful and necessary in describing and explaining many processes. To meet the challenge of their complexity, and in order to model numerically such features and capture as correct as possible these multiscale phenomena, mathematical modelling and theoretical concepts combined with the development of efficient algorithms and simulation tools must be emphasized and promoted.

### 3 Abstracts for the poster session

**Thomas Eiter (TU Darmstadt)**

*Title:* Estimates of time-periodic fundamental solutions to the linearized Navier-Stokes equations

*Abstract:* Fundamental solutions to the time-periodic Stokes and Oseen linearizations of the Navier-Stokes equations in dimension  $n \geq 2$ , this means, to the system of equations

$$\begin{cases} \partial_t u - \Delta u + \lambda \partial_{x_1} u + \nabla \mathbf{p} = f & \text{in } \mathbb{R} \times \mathbb{R}^n, \\ \operatorname{div} u = 0 & \text{in } \mathbb{R} \times \mathbb{R}^n, \\ u(t, x) = u(t + \mathcal{T}, x), \end{cases}$$

are introduced and investigated. Here  $\mathcal{T} > 0$  denotes a fixed time-period,  $\lambda \in \mathbb{R}$  is the Reynolds number, and data  $f$  with the same time-period is considered.

To model the time-periodicity, the torus group  $\mathbb{T} := \mathbb{R}/\mathcal{T}\mathbb{Z}$  is introduced and the equations are reformulated on the locally compact abelian group  $\mathbb{T} \times \mathbb{R}^n$ , where the fundamental solutions are decomposed into two parts: a *steady-state* part, which is identified with the well-known fundamental solution to the stationary Stokes and Oseen linearizations, and a *purely periodic* part, for which integrability properties and pointwise estimates are derived.

For this purpose, results from Fourier analysis on the group  $\mathbb{T} \times \mathbb{R}^n$  are applied, in particular, the so-called transference principle, which yields a method to identify Fourier multipliers on groups of this kind. For the derivation of pointwise estimates, properties of fundamental solutions to the Laplace equation and to (a modification of) the Helmholtz equation are employed.

The derived integrability properties and pointwise estimates have some immediate consequences. At the outset, the fundamental solutions are introduced as distributions on the Schwartz-Bruhat space on  $\mathbb{T} \times \mathbb{R}^n$ . However, the integrability properties ensure that they can be identified as elements of an appropriate  $L^q$  space, and convolutions with fundamental solutions can thus be expressed classically via integrals. With the help of the pointwise estimates, one can investigate the asymptotic behavior of time-periodic solutions to both the Stokes and the Oseen equations.

Reference:

- [1] Thomas Eiter and Mads Kyed. Estimates of time-periodic fundamental solutions to the linearized Navier-Stokes equations. *J. Math. Fluid Mech.*, pages 1–13, 2017. doi:10.1007/s00021-017-0332-7.

**Elfriede Friedmann (Universität Heidelberg)**

*Title:* Modeling and Simulation of the effectiveness of drugs

*Abstract:* Complex systems-biological processes are involved in the action of drugs. Often, these processes are local and include in addition to chemical interactions also a diffusive character. On the example of the treatment of retinal diseases, where the drug is injected in the vitreous, we model the distribution and action of the drug to analyze its effectiveness. Therefore, we develop a mathematical model consisting of systems of partial differential equations involving anisotropic diffusion to include the effect of the collagen fibers which have a certain orientation in the vitreous body. The simulations are performed with the Finite Element method in our Virtual Eye, a data-based realistic three-dimensional geometry model. To improve the effectiveness of the drug we analyze the position of injection by introducing specific output functionals which measure the mean or relative amount of the drug in the vitreous and in the area of action.

This is a joint work with Simon Dörsam (Institute for Applied Mathematics, IWR, Heidelberg University) and Gerd U. Auffarth (Department of Ophthalmology, David J. Apple International Laboratory for Ocular Pathology, International Vision Correction Research Centre (IVCRC), Heidelberg University).

**Hyeon Jeong Kim (Universität Heidelberg)**

*Title:* Traveling wave solutions for a thin-film equation related to the spin-coating process

*Abstract:* We study a problem related to the spin-coating process in which a fluid coats a rotating surface. Our interest lies in the contact-line region for which we propose a simplified traveling wave approximation. We construct solutions to this problem by a shooting method that matches solution branches in the contact-line region and in the interior of the droplet. Furthermore, we prove uniqueness and qualitative properties of the solution connected to the fourth-order nature of the equation, such as a global maximum in the film height close to the contact line, elevated from the average height of the film.

**Xinye Li (Universität Aachen)**

*Title:* Spectral stability and steady state dynamics of chiral skyrmions

*Abstract:* Finer mathematical properties of chiral skyrmions come into play e.g. if one asks the question whether a skyrmion can be accelerated to arbitrary velocities or whether there exists a terminal velocity beyond which the skyrmion will collapse. Crucial concepts are spectral stability estimates and topological energy bounds. In this contribution we investigate stability properties of the axially symmetric solution corresponding to the isolated chiral skyrmion in noncentrosymmetric ferromagnets. By performing a spectral analysis, we prove linear stability of the skyrmion under arbitrary perturbations in the regime where the external magnetic field is sufficiently large. As a consequence we obtain the existence of a traveling wave solution with a small in-plane spin velocity to the Landau-Lifshitz-Gilbert equation with spin transfer torque. (Based on joint work with Christof Melcher (RWTH))

**Frank Rösler (Universität Freiburg)**

*Title:*

*Abstract:* We prove norm-resolvent convergence for the Dirichlet problem of the operator  $-\Delta$  in the perforated domain  $\Omega \setminus \bigcup_{i \in 2\mathbb{Z}^d} B_{a_\varepsilon}(i)$  to the limit operator  $-\Delta + \mu$  on  $L^2(\Omega)$ , where  $\mu$  is related to the harmonic capacity of the unit ball and  $\Omega$  is an open subset of  $\mathbb{R}^d$ ,  $d \geq 2$ .

This is an improvement of previous results [Cioranescu & Murat. A Strange Term Coming From Nowhere, *Progress in Nonlinear Differential Equations and Their Applications*, 31, (1997), 45–93; J. Rauch & M. Taylor. Potential and scattering theory on wildly perturbed domains, *J. Funct. Anal.*, 18, (1975), 27–59], which show *strong* resolvent convergence. In particular, our result implies Hausdorff-convergence of the spectrum of the resolvent for the perforated domain problem.

**Zisis Sakellaris (Universität Aachen)**

*Title:* A minimization problem related to scalar curvature

*Abstract:* Let  $(M, g_0)$  a smooth compact Riemannian manifold with smooth boundary and dimension  $n \geq 3$ . We consider a minimization problem for the scalar curvature  $R$  after a conformal change. In particular, we seek for minimizers of the  $\|\cdot\|_\infty$  functional of  $R$ , within a conformal class, under small energy assumptions and natural geometric constraints. We prove that minimizers exist, and have locally constant scalar curvature, outside of a set  $\Gamma$  with explicit description. Moreover, we discuss connections of our problem with recent advances on solutions of other variational problems in  $L^\infty$ , and the so called  $\infty$ -Bilaplacian.

**Wenhui Shi (University of Coimbra)**

*Title:* Optimal regularity results for the thin obstacle problem

*Abstract:* The thin obstacle problem concerns the minimizer of the classical Dirichlet energy

$$\int_{\Omega} |\nabla u|^2 dx,$$

under the constraint that  $u$  is larger or equal to a given function  $\phi$  on a hypersurface  $\mathcal{M}$  in  $\Omega$ . Main problems of interest are the (optimal) regularity of the minimizer, and the regularity of the free boundary  $\partial\{x \in \mathcal{M} : u(x) > \phi(x)\}$ .

I will present some recent results on the optimal regularity of the solutions and the free boundaries for the thin obstacle problem under suitable regularity assumptions on the hypersurface  $\mathcal{M}$  and obstacle  $\phi$ . Techniques involved are Carleman estimates, boundary Harnack inequalities and hodograph-Legendre transformation. This is based on joint work with H. Koch and A. Růland.

**Stephan Wojtowytsch (Universität Freiburg)**

*Title:* Phase Field Models for Thin Membranes Under Topological Constraints

*Abstract:* We consider the problem of minimising a curvature energy in the class of closed, connected surfaces with given surface area that are confined to a fixed container. Based on a phase field model for Willmore's energy due to de Giorgi, we develop a technique to incorporate the connectedness constraint into a diffuse interface model of elastic membranes. Our approach uses a geodesic distance function associated to the phase field to discern different connected components of the support of the limiting mass measure.